

*Magnetic Induction and its Reversal in Spherical Iron Shells.**

By J. W. NICHOLSON, M.A., D.Sc., Professor of Mathematics in the University of London; and ERNEST WILSON, M.Inst.C.E., M.Inst.E.E., Siemens Professor of Electrical Engineering in the University of London, King's College.

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In a previous communication† to the Society, an account was given of an investigation of the magnetic shielding of a large space, by the use of a series of spherical iron shells. The basis of this investigation, which is still in progress, is strictly quantitative, and the work has been retarded, to a great extent, by the fact that several processes adopted commonly by various investigators of magnetic phenomena have never been examined accurately on the theoretical side, and in several cases even on the experimental side, to an order of precision which is sufficient for delicate measurements. One of the main objects ultimately in view, in the work described already, is the production, throughout the volume of a large iron specimen, of a magnetic field so small, in comparison with that of the earth, for example, as to render possible some accurate conclusions regarding the behaviour of iron under indefinitely small magnetic forces. This behaviour is at present a matter of doubt, and it remains to be shown that the indications of its real nature, obtained in earlier papers by one of us, continue to point in the same direction when the force is decreased below any values which have been employed hitherto. In the experiments described in the former paper, a field so low as 0.0033 C.G.S. unit was obtained in a spherical space of radius 32.5 cm. by the use of four concentric shells. It was accompanied, however, by a leakage field of 0.006 C.G.S. unit, and the shells still showed some traces of permanent polarisation after continued treatment with many reversals of a slowly decreasing current.

This method of demagnetisation is used extensively, and in its application coils have sometimes been wound somewhat loosely on the iron to be demagnetised, and the constant field given by Maxwell's formula for a spherical internal space surrounded by a very closely wound coil assumed to exist. But if a thin shell is wound in this manner, although the more internal parts may be subject to this field, the results must be widely different in the outer material of the shell. Further experimental work on

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the improvement of magnetic shielding beyond the order obtained already—in fact by such methods as the employment of a very small magnetising current to remove, in the internal space, the small part of the earth's field which may remain after the operation of the shielding effect of a set of shells—requires a precise knowledge of the value of the magnetic induction in a shell, or one member of a set of shells, wound by a spherical helical coil. For the condition of indefinitely close packing is found to be essential to the success of Maxwell's formula, which—but only from this particular point of view—does not appear, either from an *a priori* theoretical consideration, or from the experimental results described in the present paper, to have the large range of validity which has been assumed for it hitherto by practical workers on magnetic phenomena.

It is noteworthy that Maxwell himself did not claim any validity for his expression of the force inside a spherical helix when that helix was not in fact in the ideal state of indefinitely close winding. The use of such a helix for the purpose of demagnetisation of a hollow shell—where importance attaches more to the material of the shell than to the air space within—was not adopted at that time, and the practical problem had given no indication of its ultimate importance. Evidently the spacing of the helix constituting an ordinary cylindrically wound solenoid, if small in comparison with the radius of the cylinder, would not seriously disturb the uniform distribution of magnetic induction in a thin iron bar placed along its axis, but at the outer surface of a thick bar filling the helix the distribution would be widely different. We may argue from this by analogy to the spherically wound thin iron shells whose thickness is of the same order as the space between consecutive turns of the coil.

Apart from the case of a closely wound coil on a shell of non-magnetisable material, Maxwell did not consider the problem of the spherical helix in further detail. No subsequent writer has taken up the question, and it has been necessary to include in this paper some preliminary theoretical discussion of the problems whose experimental investigation has been required. The calculations relate to the ideal case of indefinitely close winding—no simple mathematical treatment of a more practical case has yet been found—and the results are compared with those found for the actual coils which were described in the earlier paper. An estimate of the effect of spacing in the helix is thus obtained, and the calculations are of importance in themselves, as indicating some quantitative relations, inherent in problems of this nature, which it would be difficult to obtain by general reasoning. The distribution of magnetic induction in a set of shells, all of which may possess magnetising helices internally and externally, follows certain very curious laws.

Into the experimental problem of reversal of the magnetic induction of a shell, two main factors enter. The first is the nature of the magnetising coil and the necessary current it must contain. As stated already, the investigation of this problem is one of the main objects of the present paper. But in addition, demagnetisation of a mass of iron is much retarded by eddy currents set up in the iron by any change applied to the magnetising forces. These currents, while flowing, shield the inner shells, in the case of a set, from the effect of the changed field, and if the reversals of current in the coils are made too quickly, the process is very inefficient as a demagnetising agency, for, in the time occupied by two reversals, the effect does not penetrate to the inner part of the iron.

The latter part of the paper contains a further contribution to the study of this phenomenon, as it is necessary to obtain some estimate of the time required between reversals, for small values of the magnetic induction. Previous work on eddy current phenomena has been almost completely restricted to cases in which the average magnetic induction is of a much higher order of magnitude than in the present experiments.

Before proceeding to a description of the experimental work, it is desirable to give some account of the solution of the theoretical problems which are involved, and which have not hitherto received attention. The analysis required throughout, however, is a fairly simple extension of a well-known method of treating a set of shells without magnetising coils, and for this reason the work is set out somewhat briefly, and, in certain of the cases, only the final results are quoted when there is no novelty in the mode of treatment or when the complete investigation is somewhat long, and only interesting from the point of view of its final results. No attempt is made to deal with other than the ideal cases in which the magnetising helices are wound extremely closely. In this manner, two simple surface conditions become applicable at every coil—the continuity of normal magnetic force, and a discontinuity of a prescribed amount in the magnetic potential—as shown in the following section. But the distribution of magnetic induction in the actual set of shells must be similar, in a general manner, to that calculated for these ideal cases, whose consideration has, in fact, served to direct the experimental work into the most fruitful channels.

The Spherically Wound Magnetising Coil.

The fact that a coil wound closely over a sphere of unit permeability, with all its turns perpendicular to the same diameter, produces a uniform internal field is well known.* For a diameter $2a$, with $2na$ turns each carrying a current i ,

* Maxwell's 'Electricity and Magnetism,' vol. 2, para. 675 (ed. 1873).

internal potential $\Omega = -\frac{8}{3}\pi inr \cos \theta$. The potential itself, on the other hand, is discontinuous at $r = a$, for

$$\Omega_{\text{out}} - \Omega_{\text{in}} = a \left(\frac{4}{3}\pi in + \frac{8}{3}\pi in \right) \cos \theta = 4\pi ina \cos \theta,$$

so that the amount of the discontinuity is $4\pi ina \cos \theta$. These constitute the two surface conditions mentioned already as applicable at the surface of the coil, and they admit of an instructive comparison with the corresponding conditions for a spherical shell magnetised normally at every point—the discontinuity of potential is in this case $4\pi\phi$, where ϕ is the strength of the shell.

We may now consider a spherical helix wound closely on a shell of non-magnetic material, enclosing a concentric shell, of inner and outer radii a and b , and of permeability μ . The radius of the coil is c , and it is wound with n turns per unit length of diameter, each carrying a current i . With the origin at the centre, and the diameter perpendicular to the windings as initial line of polar co-ordinates, the field in the hollow of the inner shell may be defined by a potential $\Omega_1 = Ar \cos \theta$, whereas in the magnetic material of the inner shell the potential is of the form

$$\Omega_2 = (Br + C/r^2) \cos \theta.$$

Capital letters denote constants as usual. Between this shell and the coil, and outside the coil, the potentials are respectively

$$\Omega_3 = (Dr + E/r^2) \cos \theta, \quad \Omega_4 = F/r^2 \cdot \cos \theta.$$

The usual conditions are fulfilled at the two surfaces of the shell of permeability μ , thus

$$A = B + C/a^3 = \mu(B - 2C/a^3), \quad B + C/b^3 = D + E/b^3, \\ \mu(B - 2C/b^3) = D - 2E/b^3.$$

These four equations may be supplemented by the conditions at the coil, where $r = c$. These are

$$\partial\Omega_3/\partial r = \partial\Omega_4/\partial r \quad \text{and} \quad \Omega_4 - \Omega_3 = 4\pi inc \cos \theta.$$

Accordingly

$$D - 2E/c^3 = -2F/c^3, \quad D + E/c^3 = -4\pi in + F/c^3,$$

whence

$$D = -\frac{8}{3}\pi in, \quad C = -\frac{8\pi ina^3(\mu-1)}{9\mu+2(\mu-1)^2(1-a^3/b^3)}, \\ B = -\frac{8\pi in(2\mu+1)}{9\mu+2(\mu-1)^2(1-a^3/b^3)},$$

from the solution of the six equations. These values are, as might be expected, independent of the radius of the magnetising coil, which remains

equivalent, in its effect on an inside iron shell, to a uniform magnetic field external to the shell.

The most convenient magnitude for measurement in such a thin shell is the maximum transverse induction (transverse to the radius vector at $\theta = 90^\circ$) when the use of an exploring coil is adopted, as in succeeding experiments. Now, the magnetic potential in the material is

$$\Omega_2 = (Br + C/r^2) \cos \theta,$$

and the transverse magnetic induction is $-\mu \partial \Omega_2 / r \partial \theta$ or $-\mu (B + Cr^{-3}) \sin \theta$. It is greatest in the plane $\theta = 90^\circ$, and the average value in that plane, denoted subsequently by $B_{\max.}$, is

$$B_{\max.} = -\frac{2\mu}{b^2 - a^2} \int_a^b (B + Cr^{-3}) r dr = \mu B + \frac{2\mu C}{ab(a+b)}.$$

For a thin shell, the variation across the section $\theta = 90^\circ$, or any other section, is so small that $\mu (B + Ca^{-3})$ may be used for $B_{\max.}$, so that, quoting the values of B and C , we obtain

$$B_{\max.} = -\frac{24\pi in\mu^2}{9\mu + 2(\mu - 1)^2(1 - a^3/b^3)}.$$

If $1 - a^3/b^3 = 3\epsilon$, where, in the subsequent experiments, ϵ is about $\frac{1}{15}$,

$$B_{\max.} = -8\pi in\mu / (3 + 2\mu\epsilon)$$

to a good approximation.

It is, perhaps, desirable at this point to lay some emphasis on the smallness of the values of $B_{\max.}$ produced in a single shell by the demagnetising currents adopted in the experiments described in the former paper. For $\epsilon = \frac{1}{15}$, $n = \frac{1}{2}$, and $\mu = 150$, a current of 1 ampère, or 10^{-1} C.G.S. unit—which was the order of the magnitude of the current used—only produces an average magnetic induction, in a single shell, of amount

$$\frac{4}{15}\pi \times 150/23 = 8.5 \text{ C.G.S. units,}$$

at $\theta = \pi/2$. The actual value will be even smaller, on account of the loose winding of the coil, which corresponds to $n = \frac{1}{2}$.

Consider now a magnetising coil, of radius c , wound internally to the shell of radii a and b , perhaps on a shell of non-magnetic material in the interior. The fields in the various regions are represented very simply. In the space internal to the coil the field is uniform, whereas between the coil and shell, and in the material of the shell, it can in each case be represented by the superposition of a uniform field and that of a central magnetic doublet of suitable moment. Outside the shell, being finite at infinity, it is the field of a central magnetic doublet only. There is no necessity to record the

analysis, which follows the same lines as before. The final magnetic potential in the material of the shell is found to be

$$\Omega = (Ar + B/r^2) \cos \theta,$$

where

$$A = \frac{8\pi in(\mu-1)c^3/b^3}{9\mu+2(\mu-1)^2(1-a^3/b^3)}, \quad B = \frac{4\pi inc^3(\mu+2)}{9\mu+2(\mu-1)^2(1-a^3/b^3)}.$$

The average value of $B_{\max.}$, obtained as before, is

$$B_{\max.} = \mu A + 2\mu B/ab(a+b),$$

or, to a close approximation for a thin shell,

$$B_{\max.} = \frac{12\pi in\mu^2 c^3/b^3}{9\mu+2(\mu-1)^2(1-a^3/b^3)}$$

directly proportional to the cube of the radius of the coil. If the coil is effectively wound on the inner surface of the thin shell of magnetic material, we may put $c = a$, and comparison with the previous case indicates an interesting theorem:—

The maximum transverse magnetic inductions obtained by winding a thin iron shell (1) internally, and (2) externally, with the same coil carrying the same current are in the ratio 1 : 2.

No conclusion, however, can be drawn from this result as regards the relative maxima induced simultaneously in two thin shells of nearly equal radii with a coil wound in the small air space between them. Their mutual effect is then of great importance and is considered in the next section.

Two Concentric Shells enclosing a Magnetising Coil.

The analysis of the arrangement is somewhat long, but it follows the previous lines so closely that we only present a bare summary. Let (a, b) and (d, e) be the two radii of the inner and outer shells, each of permeability μ , and let c be the radius of the coil situated between them, so that a, b, c, d, e are in ascending order of magnitude. Between $r = a$ and b , b and c , c and d , and d and e , two constants are in each case required to define the magnetic potential when a current is passed through the coil. When $r = a$ the field is uniform, and beyond $r = e$ it is that of a central magnetic doublet. Each of these two regions requires one constant, so that, for a complete specification of the magnetic distribution produced by the current, 10 constants must be determined. The usual conditions at the four surfaces of the shells lead to eight equations, and two other equations are derived from the continuity of normal force, and the discontinuity of potential, $4\pi inc \cos \theta$, at the coil $r = c$. The equations are as usual linear, and their final solution is expressed most conveniently as follows:—

If

$$P = 2(\mu-1)(\mu+2)(1/d^3-1/e^3)/\{9\mu+2(\mu-1)^2(1-d^3/e^3)\},$$

$$Q = (\mu-1)(2\mu+1)(b^3-a^3)/\{9\mu+2(\mu-1)^2(1-a^3/b^3)\},$$

both being positive, the transverse magnetic induction at $\theta = 90^\circ$ is, at the inner surface of the inner shell,

$$B_{\max.} = -\frac{12\pi in\mu^2(c^3P+2)}{(1-PQ)\{9\mu+2(\mu-1)^2(1-d^3/e^3)\}},$$

and at the outer surface of the outer shell,

$$B_{\max.} = \frac{12\pi in\mu^2(2Q+c^3)/e^3}{(1-PQ)\{9\mu+2(\mu-1)^2(1-d^3/e^3)\}}.$$

These may be taken as the average values across the sections of maximum, in the case of the shells to which our experiments relate, with an error of not more than about 3 per cent. In the special cases $d = e$, whence $P = 0$, and $a = b$, whence $Q = 0$, one shell is abolished, and the two previous solutions for a single shell are obtained. This solution is exact when the coil is wound indefinitely closely. It is of interest to calculate the numerical values in one special case for future comparison, and for this purpose we select the two largest shells used in the experiments, and referred to in the former paper as Nos. 3 and 4. Their permeabilities were practically identical, and $\mu = 148.5$. The radii of the inner (No. 3) were 38.5 and 40.5 cm., and of the outer (No. 4) 42.5 and 44.5 cm. Let the coil be wound on the inner so that $c = 40.5$ cm., then

$$\mu = 148.5, \quad a = 38.5, \quad b = 40.5 = c, \quad d = 42.5, \quad e = 44.5,$$

whence

$$P = 0.041055, \quad Q = 54675, \quad PQ = 0.5766, \quad 2+c^3P = 2.701, \\ (c^3+2Q)/e^3 = 1.995, \quad 9\mu+2(\mu-1)^2(1-d^3/e^3) = 7019.7,$$

and therefore, when excited by a current i in n turns per unit diameter of a coil wound on the inner shell, $B_{\max.}$ in the outer should be

$$B_{\max.} = 558.0 in$$

on reduction, and in the inner,

$$B_{\max.} = -755.5 in,$$

their ratio being (inner)/(outer) = 1.345. The mutual action of the shells has thus changed the ratio from 1:2 to 3:4. The maxima are therefore more nearly equal than might have been anticipated, and, in the earlier experiments, a current used to demagnetise any one shell must have been almost equally effective in the case of the shell on the other side of the air

space containing the coil. Moreover, in the absence of the outer shell, the value of $B_{\max}$, produced in the inner may be shown to be

$$B_{\max.} = 236in,$$

so that its value is increased in the inner shell, by the presence of the outer, in the ratio 3·2 : 1. These results may be summarised as follows for this pair of shells:—

If a coil wound round the inner, in the absence of the outer, produces a value of $B_{\max}$, equal to unity, then (1) if the inner is removed, and the outer placed round the coil, $B_{\max}$, in the outer is $\frac{1}{2}$, and (2) if both are present simultaneously, $B_{\max}$, in the inner is about 3·2, and in the outer is 2·4. We can deduce that in the demagnetisation of the entire set of shells in the original experiments, the application of currents of alternate signs simultaneously in all the coils must be at least six times as effective, for the second and third internal shells, as the successive application of these currents—a result which could hardly have been foreseen from general principles. Certain minor features in the curves then obtained can be interpreted on this basis.

These ratios necessarily become larger in the experiments described below, when all four shells are present simultaneously, and the current is passed through the coil on No. 3. But we do not include a detailed calculation of this case, which would be tedious. A discussion of one or two particular problems can be obtained somewhat simply, and, by comparison with experiment, will enable a determination to be made of the effect to which the theory may be vitiated by loose winding of the coils. We shall accordingly consider only this special problem to which the more interesting experiments relate.

Magnetising Coil applied to the Outermost of a Set of Four Shells.

The most important experiment—with reference to the testing of the theory in its ultimate application to the further shielding of the internal hollow of a set of four shells by the use of a magnetising current—is perhaps that in which the current is applied to a coil wound round the outer shell, whose value of $B_{\max}$, is tested by an exploring coil, as described later. The necessary calculation for this case is greatly facilitated by the use of the recurrence formulæ developed in the earlier paper,* whose notation will be adopted without further description. The constant field in the hollow being u_1 , calculation gives the following successive values of the functions u and v there described.

The mean radii of the four shells and air space form the sequence

$$30\cdot5, 32\cdot5, \dots, 42\cdot5, 44\cdot5 \text{ cm.},$$

* ‘Roy. Soc. Proc.’ A, vol. 92, p. 535 (1916).

and the permeabilities, from inner to outer, are 82.4, 163, 148, 148.9—where fairly large error in the first, and a smaller error in the second, are of no importance. We then find, applying the recurrence formulæ without a rougher approximation than 1 per cent.,

$$\begin{aligned}\epsilon_1 &= \frac{1}{3} \{1 - (30.5/32.5)^3\} = 0.0578, & \epsilon_{12} &= \frac{1}{3} \{1 - (32.5/34.5)^3\} = 0.0547, \\ \epsilon_2 &= 0.0518, & \epsilon_{23} &= 0.0493, & \epsilon_3 &= 0.0470, & \epsilon_{34} &= 0.0449, \\ 2\mu_1\epsilon_1\epsilon_{12} &= 0.5217, & 2\mu_2\epsilon_2\epsilon_{23} &= 0.8529, & 2\mu_3\epsilon_3\epsilon_{34} &= 0.6246, \\ 2\mu_1\epsilon_1 &= 9.537, & 2\mu_2\epsilon_2 &= 17.301, & 2\mu_3\epsilon_3 &= 13.910, \\ \alpha_1 &= 1.4092, & \alpha_2 &= 1.7518, & \alpha_3 &= 1.5367, \\ \beta_1 &= 0.0547, & \beta_2 &= 0.0493, & \beta_3 &= 0.0449, \\ \gamma_1 &= 8.603, & \gamma_2 &= 15.694, & \gamma_3 &= 12.294, \\ \delta_1 &= 0.775, & \delta_2 &= 0.798, & \delta_3 &= 0.824.\end{aligned}$$

It is desirable that these intermediate functions should be placed on record for reference in future work on magnetic shielding, which is now in progress with these specimens of iron.

Again,

$$\begin{aligned}u_2/u_1 &= 1.464, & v_2/v_1 &= 9.378, \\ u_3/u_1 &= 3.027, & v_3/v_1 &= 30.46, \\ u_4/u_1 &= 6.020, & v_4/v_1 &= 62.32,\end{aligned}$$

and finally $v_4/u_4 = 10.35$,

relating to the air space just inside the fourth shell, on which the coil is wound. If the potential in the fourth shell is

$$\Omega = (Ar + B/r^2) \cos \theta,$$

and its inner and outer radii are a and b , accordingly

$$\mu(A - 2B/a^3) = 10.35(A + B/a^3),$$

giving the ratio of A to B , where μ is its permeability. Application of the conditions at $r = b$ and at the coil shows, moreover, that

$$-8\pi in = A(\mu + 2) - 2(\mu - 1)B/b^3,$$

so that A and B may be obtained. Thus in the outer shell,

$$B_{\max.} = \mu(A + B/b^3) = -\frac{8\pi in\mu \{1 + [(\mu - 10.35)/(2\mu + 10.35)]a^3/b^3\}}{\mu + 2 - 2(\mu - 1)[(\mu - 10.35)/(2\mu + 10.35)]a^3/b^3},$$

with $a = 42.5$, $b = 44.5$ $\mu = 148.9$,

$$B_{\max.} = 148.7 in.$$

If the current is in ampères, division by 10 is necessary in order to obtain the value in C.G.S. units.

If $n = \frac{1}{2}$, or half a turn per centimetre of diameter, were a sufficiently

close winding to render the formulæ exact, a current of 1 ampère should give the experimental value

$$B_{\max.} = 14.87/2 = 7.43 \text{ C.G.S. units}$$

for the outer shell when examined by an exploring coil.

Returning to the recurrence formulæ, we notice that $v_3/u_3 = 10.06$. This result enables us to correct the previous formula, applicable to a coil wound between the third and fourth shells, for the presence of the other two shells in the interior. For, in that formula, the function P remains unaltered and the only effect of the two inner shells is to alter Q, which becomes

$$Q = \frac{a^3 \{(\mu-1)[(2\mu+10.06)/(\mu-10.06)] b^3/a^3 - (2\mu+1)\}}{(\mu+2)[(2\mu+10.06)/(\mu-10.06)] b^3/a^3 - 2(\mu-1)}$$

instead of its original value in the former calculation, which was

$$Q = (\mu-1)(2\mu+1)(b^3-a^3)/\{9\mu+2(\mu-1)^2(1-a^3/b^3)\}.$$

The rest of the formula is as before, and (a, b, μ) refer to the third shell. The numerical value of Q in the present case is found to be $Q = 58,908$, corresponding to $a = 38.5$, $b = 40.5$, $\mu = 148.9$, and thus

$$(e^3 + 2Q)/e^3 = 2.090, \quad 1 - PQ = 0.3785,$$

and, finally, for a coil carrying a current i , wound round the third of a set of four shells of the present constants, $B_{\max.}$ in the outer is given by

$$B_{\max.} = 653.9 in,$$

or $65.39 in$, if the current is in ampères. For $n = \frac{1}{2}$, the value per ampère of current is 32.69 C.G.S. units. A comparison with a direct experiment is made later. In the meantime, it is instructive to notice that, in the absence of the two inner shells, $B_{\max.}$ for the fourth shell when the third is wound was calculated previously as $558 in$. The inner shells have raised its value in the ratio 13:11—a much smaller mutual effect is involved, as might be expected, owing to the shielding influence of the third shell.

Experimental Determination of $B_{\max.}$ by an Exploring Coil.

The four shells remained, throughout the experiments, set up concentrically, each being wound with a magnetising coil, and the axis of all the coils was the diameter coinciding with the direction of the earth's field. The value of n was in each case $\frac{1}{2}$, the number of turns wound on the shells being respectively 34, 38, 42, and 46. The various dimensions have been specified already, and the shells are formed from high permeability magnet steel. Since the previous experiments were made, the butting surfaces of all the shells have been machined in such a manner that each of the two halves of any

original shell penetrates the other. Preliminary tests, not described in this paper, have indicated that the leakage originally present on account of this lack of machining has almost entirely disappeared, so that the theoretical formulæ are strictly applicable if the coils are sufficiently closely wound.

The four concentric shells were placed with their junction planes at right angles to the direction of the earth's magnetic field, and therefore parallel to the plane of any winding of a coil. The exploring coil lay in a plane parallel also to these junction planes, and distant about 7 cm. from the junction plane of the fourth shell. It surrounded the iron of the outermost shell, referred to as No. 4, and consisted of one complete turn of insulated copper wire touching the outside of the shell, and one complete turn touching also the inside. The ends of the inside turn were brought out of the shell through a small hole in the junction plane, by bending the two ends, which are now stranded together, at right angles to the plane of the coil and then threading them through the hole. The inside and outside turns were then placed in series so that the coil completely surrounded the iron once. The section of the effective part of the coil is thus a plane annulus whose radii are practically those of the outermost shell, and the coil is cut normally, in view of its parallelism to the magnetising coils, by the transverse magnetic induction at $\theta = \pi/2$ in the outer shell, over the whole area of section of that shell. The normal induction across the coil is thus the product of the sectional area $\pi(r_1^2 - r_2^2)$, where r_2 and r_1 are the radii 42.5, 44.5 cm., of the outer shell, and the average value of $B_{\max.}$ for the outer shell as calculated already.

More strictly, this product should be multiplied by a magnitude of the order $\cos 10^\circ$, on account of the small displacement 7 cm. of the plane of the exploring coil from the diametral plane $\theta = \pi/2$. But this would only make a change not exceeding 2 per cent. The exact position of the exploring coil was so chosen that, while giving B with this degree of accuracy on one ground, it should at the same time be so far removed from the junction that the effect of any small air space should be negligible. For although the two hemispheres constituting the whole shell were dovetailed together, the degree of fit was not sufficient to ensure perfect contact throughout. A very small air space does in fact still exist, and we may therefore assume that the observed magnetic induction might on this account be slightly smaller than the calculated value even in the ideal state of packing of the magnetising coils.

The resistance of the exploring coil was quite negligible in comparison with that of the ballistic galvanometer. This instrument, which was of the d'Arsonval type, had a resistance of 467 ohms at 10°C. ; to this additional resistance was added in order to obtain the sensibility necessary to ensure

that the deflection was not of a magnitude capable of serious error on account of permanent set in the suspending fibre. Let R be the resistance of the circuit of galvanometer, exploring coil, and extra resistance when this was used, and T the periodic time of the swinging coil of the instrument, in which a steady current of C ampères produces a deflection α . Let $1+r$ be the ratio of two successive deflections of the galvanometer coil when on open circuit. Then if m is the number of turns in the exploring coil, A the area of cross-section of the iron in the plane of the exploring coil, in square centimetres, and Δ the mean of the first and second deflections obtained by reversal of the current in the magnetising coil, we can deduce* the formula :—

$$B_{\max.} = (1+r/2)(CT/2\pi\alpha)(R\Delta/2mA) \times 10^8.$$

The following values of the constants were obtained as the result of a calibration of the instrument and shell :—

$$\begin{aligned} 1+r/2 &= 1.03, & C/\alpha &= 0.685 \times 10^{-9}, \\ T &= 8.15 \text{ sec.} & A &= 562 \text{ sq. cm.} \end{aligned}$$

with $m = 1$. Throughout the experiments, double deflections were observed on a scale having $\frac{1}{2}$ mm. divisions at a distance of 1340 mm. from the mirror.

The decrement factor $1+r/2$ was used so long as it did not exceed 1.03. But the resistance of the exploring coil is so small in comparison with that of the galvanometer that, when no extra resistance is inserted in the circuit, the galvanometer is short-circuited when used for observing deflections produced by reversal of current in a magnetising coil. Under these circumstances the decrement is very much greater, and the spot of light, after the first throw, returns to zero and gives a relatively small deflection on the other side. The ordinary treatment of damping is not then applicable. But we notice that failure to correct for a damping factor 1.03 only introduces an error of about 3 per cent., and this occurs in practice in our experiments when a resistance of 10,000 ohms is added. The interpretation of readings with smaller resistances added can be found by a preliminary experiment, which was made as follows :—

The third magnetising coil, wound between the two outermost shells, was used and a current of 1.98 ampères passed through it. Deflections were observed with varied additional resistances in the circuit of the exploring coil. Reversal of current in the magnetising coil in every case produces the same value of $B_{\max.}$ in the fourth shell, and therefore the same induced electromotive force in the exploring circuit. The product of deflection and total resistance should therefore be the same in every case, if the damping factors

* See, *e.g.*, 'Phil. Trans.,' A, vol. 180, p. 445 (1889).

are identical. This product may be taken as correct with 10,000 ohms added to the resistance of the galvanometer, and thence the true deflection, in the absence of damping, calculated in the other cases. The interpretation of a reading with any added resistance may then be obtained by interpolation. This method has the advantage of assuming no theory of the abnormal damping.

The results are shown in the annexed Table:—

Table I.—Variation of Damping with Resistance.

Total resistance (ohms).	First swing.	(Product)/100.	Proportional increase necessary for coincidence.
10,467	80	8,384	1
5,467	148	8,110	1·03
1,467	428	6,334	1·32
967	566	5,547	1·51
467	804	3,859	2·17

If 1·03 is multiplied by the value in the last column, we get a practical determination of the decrement factor, independently of any theory, and valid even for the enormous factor which occurs with no added resistance. Interpolation is not in fact necessary, for all the experiments later are performed with added resistances of 0, 1000, or 10,000 ohms. In the experiments of the earlier paper, large resistances were always used, so that no discussion of abnormal damping was then necessary. Thus any first swing with 0, 1000, 10,000 ohms added resistance must be multiplied by the factors (1·03)(2·17, 1·32, 1), or 2·23, 1·36, 1·03 respectively, in order to obtain the undamped reading. We do not attempt to discuss any theory as to the origin of these values, for it constitutes an independent problem of some difficulty, and is not strictly relevant to the present paper.

Experimental Results.

The values of $B_{\max}$ have been determined in the following cases:—

(a) The four shells were set up as described, and the readings taken in correspondence with various currents in the magnetising coil on the third shell, the exploring coil being on the fourth shell.

(b) Similar readings were taken when the system was excited instead by the magnetising coil on the fourth shell.

The values of $B_{\max}$ were obtained in accordance with the discussion in the preceding section, and are shown in the subjoined Tables II and III.

Table II.—Magnetising Coil on the Third Shell.

Current in ampères.	Deflection in scale divisions.	Added resistance in ohms.	B _{max.} in C.G.S. units.	B _{max.} /current.
0·0327	9	0	0·752	23·0
0·109	36	0	2·97	27·2
0·262	90	0	7·42	28·3
0·436	169	0	13·9	31·9
0·785	325	0	26·8	34·1
1·3	280	1,000	45·9	35·3
2·1	90	10,000	76·8	36·5
3·32	167	10,000	143	47·6
5·5	317	10,000	272	49·4

Table III.—Magnetising Coil on Fourth Shell.

Current in ampères.	Deflection in scale divisions.	Added resistance in ohms.	B _{max.} in C.G.S. units.	B _{max.} /current.
0·12	11	0	0·908	7·16
0·41	41	0	3·38	8·2
0·714	78	0	6·44	9·10
5·1	58	10,000	49·5	9·7
7·1	82	10,000	70·0	9·9

Discussion.

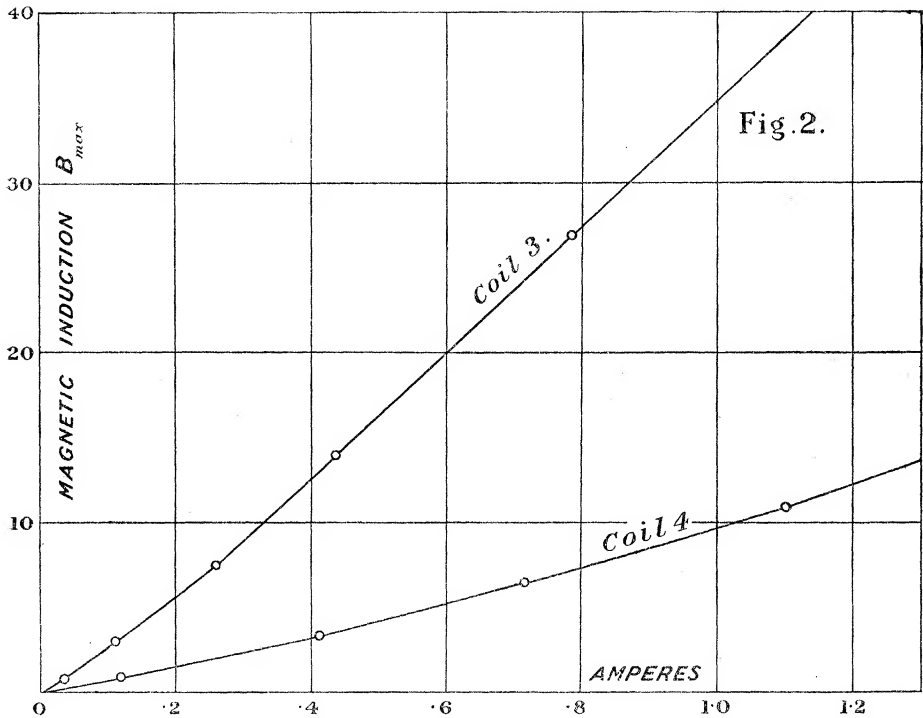
The graphs shown in fig. 2 indicate also the results of these experiments. For small values of the currents, they are very closely represented by straight lines, as would be expected. The effect of the curve of magnetisation then comes into play, so that, although, on account of loose winding of the coil, the values of B_{max.}/current are initially smaller than those predicted on theoretical grounds, they ultimately become larger than those calculated on the assumption of constant permeabilities of the shells, and the graphs become more parabolic.

The calculations of an earlier section indicate that, if the permeabilities were constant, B_{max.}/current should be equal to 32·7 when the third magnetising coil is used, and 7·4 when the current is passed through the fourth. For the smaller values of the current, the values of B_{max.} may be expected, in view of the curve of magnetisation, to follow the empirical law,

$$B_{\max.} = i(A + Bi),$$

where i is the current, at least roughly, although the constant B would have no simple physical meaning on account of the different magnetising forces on the various shells which shield one another. For currents $i = 0·1$ to $i = 1·3$ ampères, the average value of A is 31·4, neglecting the reading

corresponding to $i = 0.03$, in which an error of a scale division would be very important. This is very nearly the theoretical value, but is high on



account of initial slope of the magnetisation curve. We may take 27.2 as the true value of A very closely, when $i = 0.1$ ampère. The first four readings may be fitted into the formula

$$B_{max} = i(26.2 + 10i),$$

with errors in B_{max}/i of magnitudes $+0.1$, -0.5 , -1.3 , -0.1 . The later values require an extra term in the empirical formula, but for the currents of this order, only a very small experimental error not exceeding about 3 per cent. is indicated. Small magnetic inductions can thus be measured with extreme accuracy by this method of the exploring coil. The limiting value of the ratio with the present magnetising coil is evidently about 26.2 instead of 32.7, showing that Maxwell's formula involves a considerable error when applied to a coil even so closely wound as the present one, when the iron of the core is near the coil. At the same time, the data are now available for future use of these coils, and the need for a theoretical investigation of a loosely wound coil, which has been prominent for some time, without any solution, is removed. The Table for the coil on the fourth

shell exhibits the same general characteristics, and calls for no further remarks.

Perhaps one of the most interesting problems opened up by the present investigation is an enquiry into the limit to which demagnetisation of a shell can be carried. It would be necessary, for certain types of previous magnetic history of the iron, to apply reversals of a continually diminishing force, which began with a value corresponding to maximum permeability, as shown on the (BH) curve of the material. This would be difficult, on account of the necessary magnitude of the current for an arrangement of shells such as the present, which, however, as shown in the earlier paper,* is that of maximum shielding for a large space. Since effective demagnetisation, or, at least, the production of uniformity in the internal field of the central hollow, is a necessary preliminary to the removal of that residual field by the application of currents, the problem is of some importance. But at this point it is sufficient to indicate the necessity for a determination of what constitutes "effective demagnetisation."

Induced Currents.

The reversal of magnetisation within undivided iron magnet cores has been studied to a great extent on its experimental side, and it is known that the induced currents in the iron continue for a considerable time, and tend to maintain the magnetisation in its condition preceding the reversal. When a ballistic galvanometer is used in combination with an exploring coil wound on such magnets, the deflections of the instrument do not give the complete change of magnetic induction; the effect is similar to the damping caused by connecting a resistance across the terminals of the instrument. A continued current in one direction, caused by the slow change of magnetisation, gives rise to a first deflection considerably greater than the second—in the opposite direction—and the third deflection may exceed the second.

So far as we are aware, the lowest forces at which the delay caused by induced currents in large masses has been examined, are $1.7\ddagger$ and $1.2\ddagger$ in C.G.S. units. In the first case the magnet was of wrought iron of diameter 10.16 cm. (4 inches), and in the second, of dynamo magnet steel of diameter 30.5 cm. (12 inches). Changes at the centre of the core were observed 46 seconds after reversal in the former case and 400 seconds in the latter. The experiments showed, moreover, that, with small forces of this order of

* 'Roy. Soc. Proc.,' A, vol. 92, p. 529 (1916).

† 'Phil. Trans.,' A, vol. 186, p. 93 (1895).

‡ 'Journ. Inst. Elec. Eng.,' Pt. 116, vol. 24, p. 194.

magnitude, there is but a single maximum rate of change of the magnetic induction, which occurs at a comparatively early stage.

But the values of the magnetic induction corresponding to these forces were of the orders 7000 and 5000 C.G.S. units. In the experiments of the present communication, the magnetic induction in the outermost shell is only 67·6 C.G.S. units when a current of 7 ampères is used in its magnetising coil. Corresponding to a current of 5·5 ampères in the coil wound on the third shell, the maximum transverse induction in the fourth or outermost shell is 272 C.G.S. units. These currents have not as yet been greatly exceeded in our experiments on demagnetisation. It is interesting, and of some importance, to enquire whether the delay caused by the reversal of such small inductions is sufficiently great to affect the decrement, as ordinarily obtained, of the readings of a galvanometer whose periodic time is 8 seconds.

Some experiments have been made in order to test this point. A list of deflections due to a reversal of a current in the third coil, and its corresponding effect on an exploring coil wound on the fourth shell, has been used already for another purpose. The same arrangement, with a current of 1·98 ampères reversed, and various additional resistances inserted in the galvanometer circuit, was used also in determining the ratio of first to second deflection corresponding to each resistance. The exploring coil was then cut out of the galvanometer circuit, and deflections were observed corresponding to the same resistances as before, ranging from 0 to 10,000 ohms, when the coil of the galvanometer was initially set in motion by a primary cell of low voltage working through a very high resistance, the battery circuit being open while the readings were taken. The ratios of first to second deflection were again calculated, and the comparison is made in Table IV.

Table IV.

Additional non-inductive resistance in galvanometer circuit.	Deflection ratio when caused by exploring coil constantly in circuit.	Deflection ratio when coil of galvanometer is set in motion by primary cell.
ohms.		
10,000	1·08	1·10
5,000	1·29	1·29
1,000	2·27	2·25
500	3·50	3·68
0	19·6	57·4

When the additional resistance was zero, or in short circuit, the second deflection was so small as to render the ratio very uncertain. But the ratios

corresponding to other resistances are in such close agreement that the delay in the change due to eddy currents must be effectively negligible. This result is of some importance, for it might have been inferred from the earlier experiments that a considerable delay was possible in a shell of thickness 2 cm. The argument would be applied as follows:—

In the distribution of induced currents in its mass a plate is comparable with a wire whose diameter is twice the thickness of the plate (*loc. cit.*). The time required to effect a complete reversal of any value of the magnetic induction in one of our shells should be of the same order as in a cylinder of diameter 4 cm. In two cylinders of different diameters, each subjected to the reversal of the same magnetic force, similar magnetic events will happen, but at times proportional to the squares of the diameters. Comparing therefore a solid shell of thickness 2 cm. with the solid magnet of diameter 10 cm., mentioned in the earlier experiments, the delay in the shell would be of order $46 \times (4/10)^2 = 7.4$ seconds. Comparison with the magnet of diameter 30 cm. gives $400 \times (4/30)^2 = 7.1$ seconds.

It is evident that if the induced currents are to disappear when the magnetic induction is of the order 6000, a considerable time must elapse between successive reversals of the magnetising force in this process of demagnetisation by reversal and diminution. But the present experiments show that for magnetic inductions not exceeding 300 C.G.S. units this necessary time interval is greatly reduced, for the delay due to induced currents is negligible in comparison with such a time as 8 seconds, the periodic time of the ballistic galvanometer employed. This has important implications in connection with work at high frequencies.

Summary.

(1) The paper contains a solution of certain problems which arise in the production of an effective magnetic shield for large spaces. These relate mainly to the effective demagnetisation of the shells of which the shield is constituted.

(2) Theoretical solutions of problems relating to the effects of indefinitely closely wound coils on various shells of such a shield are given, and compared with the experimental values for an actual coil, as determined by an exploring coil and ballistic galvanometer.

(3) The experiments supply an estimate of the deviations of Maxwell's formula, for the field inside a spherically wound helical wire, from the true values, when the spacing in the helix is of importance.

(4) A study of the necessary interval between current reversals in the process of demagnetisation has been made, and it is shown that the delay in reversal of magnetic phenomena in considerable masses of iron, due to eddy currents, is extremely small when the magnetic inductions are less than 300 C.G.S. units.

On the Dynamics of Revolving Fluids.

By LORD RAYLEIGH, O.M., F.R.S.

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So much of meteorology depends ultimately upon the dynamics of revolving fluid that it is desirable to formulate as clearly as possible such simple conclusions as are within our reach, in the hope that they may assist our judgment when an exact analysis seems impracticable. An important contribution to this subject is that recently published by Dr. Aitken.* It formed the starting point of part of the investigation which follows, but I ought perhaps to add that I do not share Dr. Aitken's views in all respects. His paper should be studied by all interested in these questions.

As regards the present contribution to the theory it may be well to premise that the limitation to symmetry round an axis is imposed throughout.

The motion of an inviscid fluid is governed by equations of which the first expressed by rectangular co-ordinates may be written

$$\frac{du'}{dt} + u' \frac{du'}{dx} + v' \frac{du'}{dy} + w' \frac{du'}{dz} = - \frac{dP}{dx}, \quad (1)$$

where

$$P = \int dp/\rho - V, \quad (2)$$

and V is the potential of extraneous forces. In (2) the density ρ is either a constant, as for an incompressible fluid, or at any rate a known function of the pressure p . Referred to cylindrical co-ordinates r, θ, z , with velocities u, v, w , reckoned respectively in the directions of r, θ, z increasing, these equations become†

$$\frac{du}{dt} + u \frac{du}{dr} + v \left(\frac{du}{r d\theta} - \frac{v}{r} \right) + w \frac{du}{dz} = - \frac{dP}{dr}, \quad (3)$$

* "The Dynamics of Cyclones and Anticyclones.—Part 3," 'Roy. Soc. Edin. Proc., vol. 36, p. 174 (1916).

† Compare Basset's 'Hydrodynamics,' § 19.